Topic

Linear Equation

Objective

To verify the conditions for consistency of a system of linear equations in two variables by graphical representation.

Linear equation

An equation of the form $ax + by + c = 0$, where a, b, c are real numbers, $a \neq 0, b \neq 0$ and x, y are variables; is called a linear equation in two variables.

$$a_1x + b_1y + c_1 = 0, \ (where\ a_1 \neq 0;\ b_1 \neq 0)$$
$$a_2x + b_2y + c_2 = 0, \ (\text{where } a_2 \neq 0;\ b_2 \neq 0)$$

(i) unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$;

(ii) no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$;

(iii) infinitely many distinct common solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Previous Knowledge Required

1. Plotting of points on graph paper.
2. If the two lines intersect at a point, then the system is consistent and has a unique solution.
3. If the two lines are parallel to each other, then the system is inconsistent and has no solution.
4. If the two lines are coincident then the system is consistent and has infinitely many solutions.

Material Required

Graph papers, fevicol, geometry box, cardboard.

Procedure

Consider the three pairs of linear equations

1st pair: $2x - 5y + 4 = 0, 2x + y - 8 = 0$

2nd pair: $4x + 6y = 24, 2x + 3y = 6$

3rd pair: $x - 2y = 5, 3x - 6y = 15$

1. Take the 1st pair of linear equations in two variables, e.g., $2x - 5y + 4 = 0, 2x + y - 8 = 0$.
2. Obtain a table of at least three such pairs (x, y) which satisfy the given equations.

For $2x - 5y + 4 = 0$

x	-2	0.5	3
y	0	1	2

For $2x + y - 8 = 0$

x	2	3	4
y	4	2	0

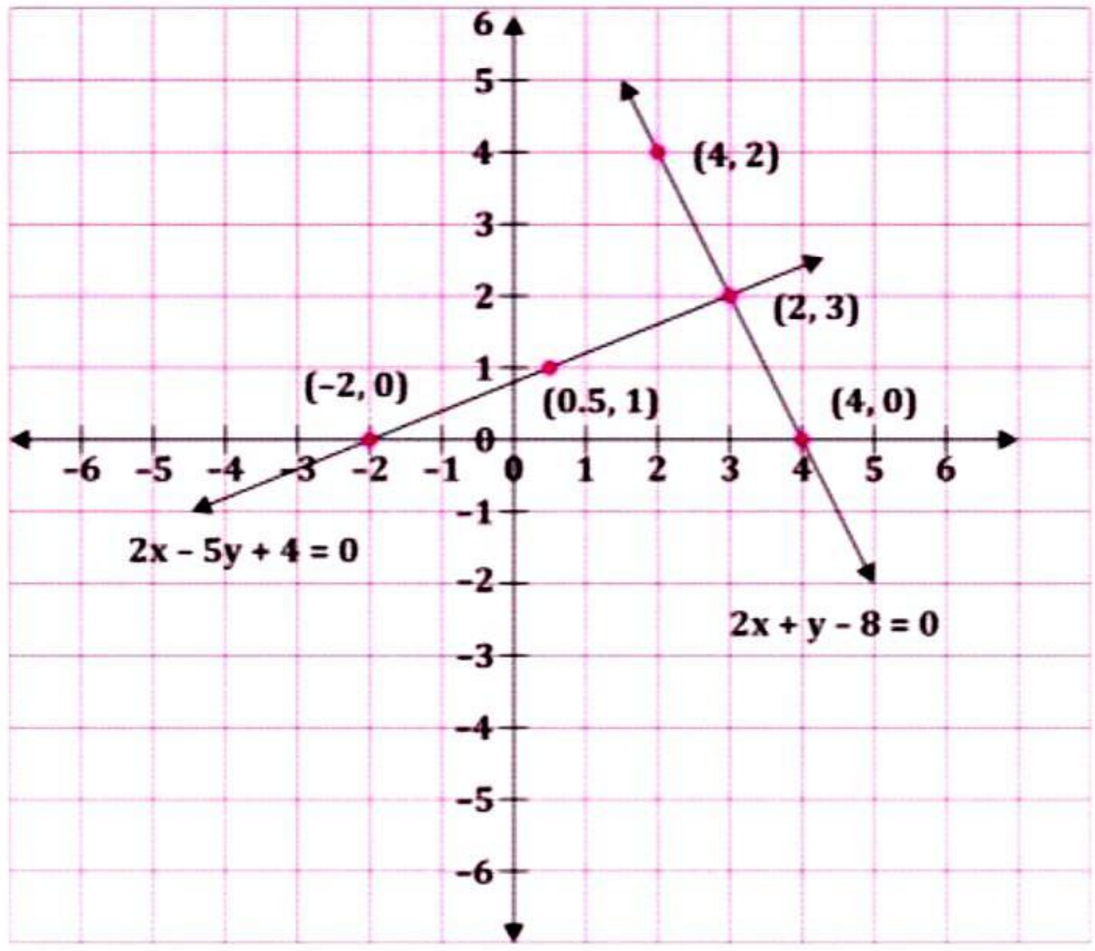

Fig.(i)

3. Plot the points of two equations on the graph paper as shown in fig.(i).
4. Observe whether the lines are intersecting, parallel or coincident. Write the values in the observation table.

   Also check, $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$
5. Take the second pair of linear equations in two variables.

For $4x + 6y = 24$

x	0	6	3
y	4	0	2

For $2x + 3y = 6$

x	0	3	1.5
y	2	0	1

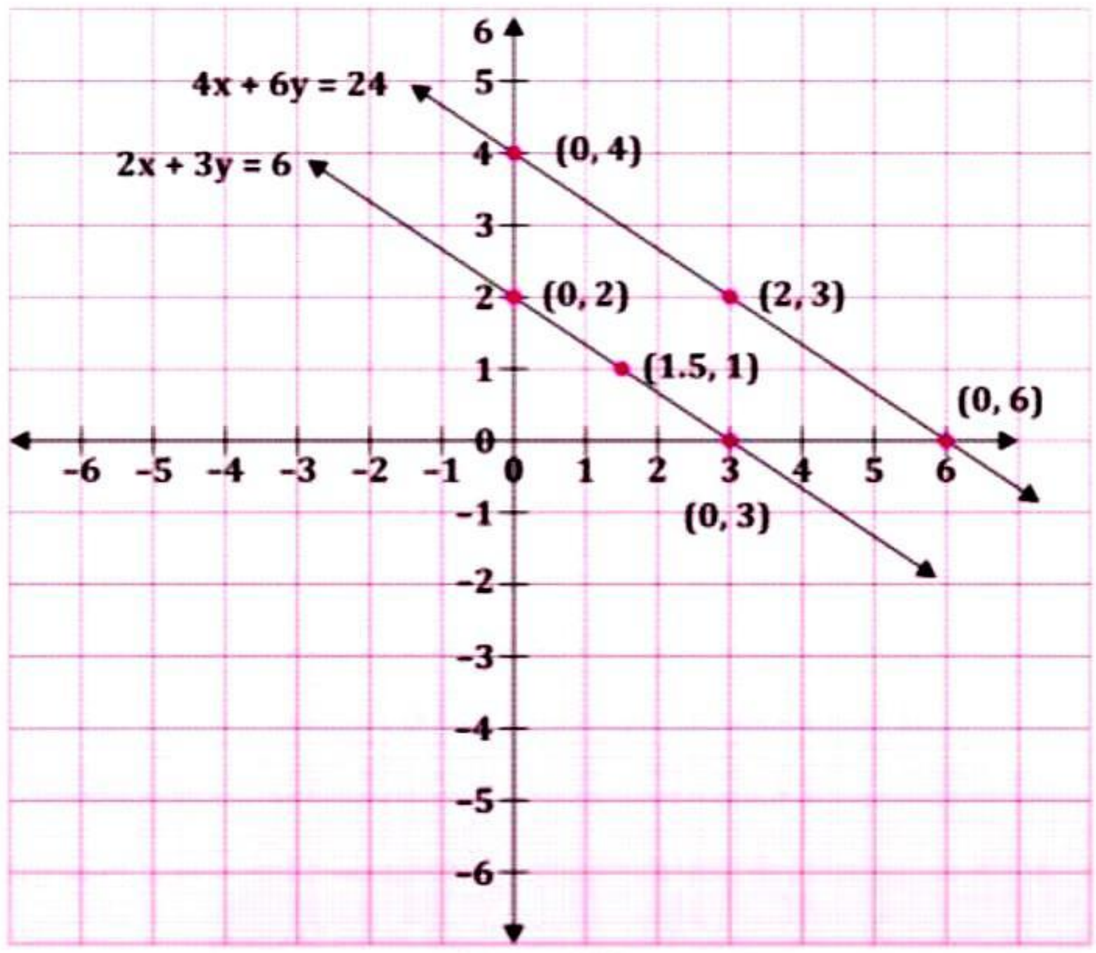

Fig.(ii)

6. Repeat steps 3 and 4.
7. Take the third pair of linear equations in two variables, i.e., $x - 2y = 5, 3x - 6y = 15$

For $x - 2y = 5$

x	1	5	0
y	−2	0	-2.5

For $3x - 6y = 15$

x	0	1	5
y	−2.5	−2	0

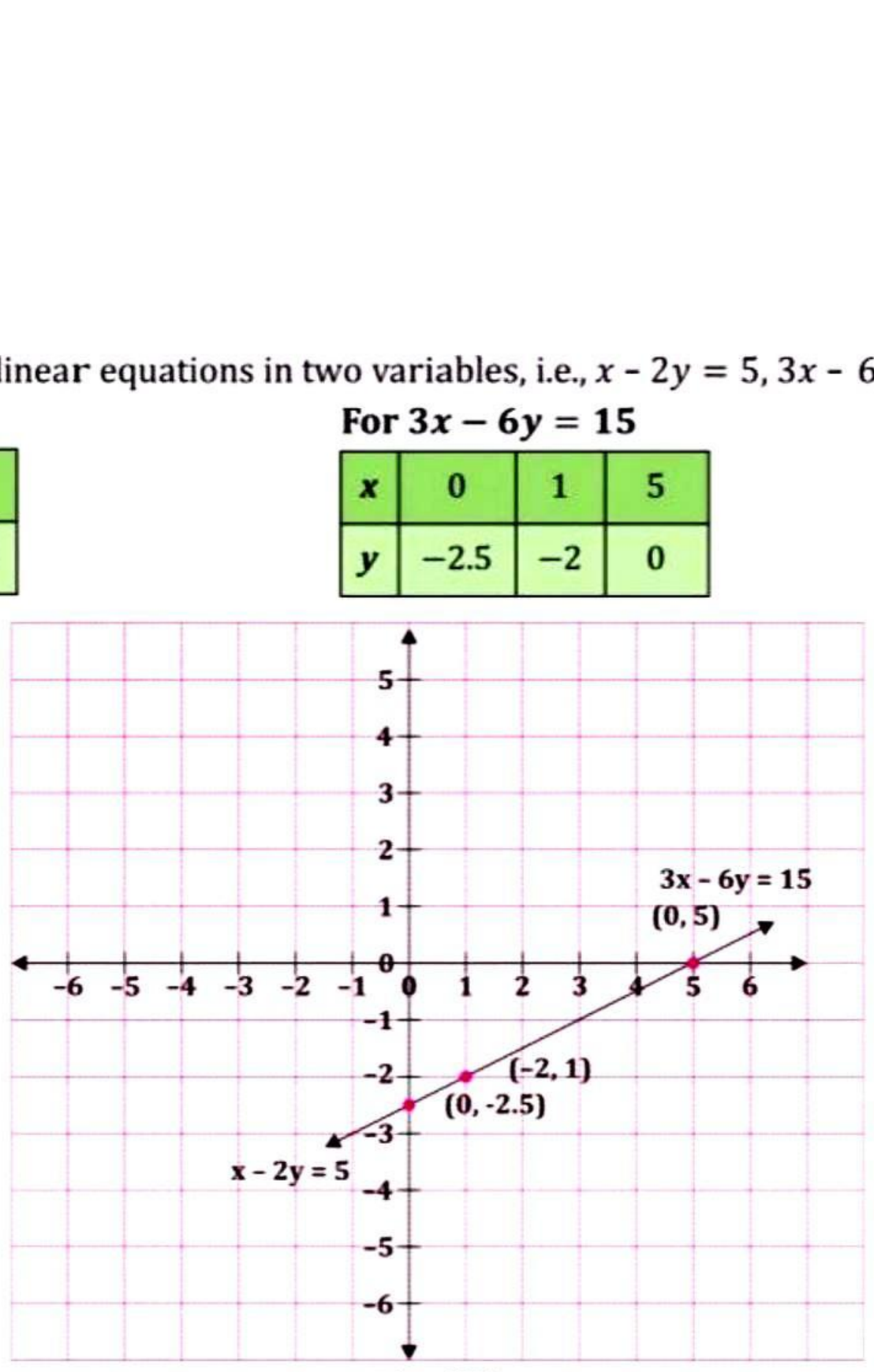

Fig.(iii)

8. Repeat steps 3 and 4.

Obtain the condition for two lines to be intersecting, parallel or coincident from the observation table by comparing the value of $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$.

Observation table					
Pairs of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	**Compare the ratios and write conditions**	**Types of lines**
1st pair					
2nd pair					
3rd pair					

Observation

Students will observe that

(i) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Result

The conditions for consistency of a system of linear equations in two variables is verified.

Learning outcome

Students will learn that some pairs of linear equations in two variables have a unique solution (intersecting lines), some have infinitely many solutions (coincident lines) and some have no solutions (parallel lines).

Activity time

Perform the same activity by drawing graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Show that there is a unique solution. Also from the graph, calculate the area bounded by these linear equations and x-axis.

VIVA VOCE

Q 1. What is the equation of a line parallel to the x-axis?

Ans. $y = a$, where a is any constant.

Q 2. What is the equation of a line parallel to the y-axis?

Ans. $x =$ b, where b is any constant.

Q 3. If $x = 0$ and $y = 0$, where would the point lie on the graph?

Ans. At origin (0,0)

Q 4. If the graphical solutions of two linear equations of two lines intersect in a plane, then what type of solution do they have?

Ans. Unique solution

Q 5. What does the graph of a linear equation represent?

Ans. A straight line

MULTIPLE CHOICE QUESTIONS

Q 1. Is $x = -1,\ y = 5$ a solution of the equation $4x + 3y = 11$?

(a) Yes (b) No (c) Can't say (d) None of these

Q 2. Equations $5x + 2y = 16$ and $7x - 4y = 2$ have:

(a) No solution (b) A unique solution (c) Infinite solutions (d) None of these

Q 3. Equations $-3x + 4y = 5$ and $\frac{9}{2}x - 6y = \frac{15}{2}$ has:

(a) A unique solution (b) Infinite solutions
(c) No solution (d) None of these

Q 4. Equations $3x + 4y = 5$ and $\frac{9}{2}x + 6y + \frac{15}{2} = 0$ have:

(a) Infinite solutions (b) A unique solution
(c) No solution (d) None of these

Answer Key

1.(a)	2.(b)	3.(a)	4. (b)